

Derivation of the tariff equivalent

As seen in McCorriston and MacLaren (2005), the trading distorting effect of the STE is defined as the level of the tariff that would be imposed on the m importing firms which would give rise to the same level of imports generated by the STE.

In mathematical terms, the relation is given by:

$$Q_m(t^e) = Q_m^{STE} \quad (\text{A-1})$$

A benchmark case is based on the following assumptions and conditions. First of all, assumption of a Cournot competition among n firms and domestic inverse demand function makes:

$$P = a - b(Q^d + Q^m) \quad (\text{A-2})$$

where $Q^d = nq^d$ (goods sourced from the domestic upstream sector), and $Q^m = nq^m$ (goods sourced from imports)

Domestic cost function and cost function for import goods are respectively:

$$P_A = f + kQ^d \quad (\text{A-3})$$

$$P_W = F + kQ^m \quad (\text{A-4})$$

Firm i 's profit function is:

$$\pi_i = (P - P_A)q_i^d + (P - P_W - t^e)q_i^m \quad (\text{A-5})$$

Specifically,

$$\pi_i = \left[a - b \left(\sum_{i=1}^n q_i^d + \sum_{i=1}^n q_i^m \right) - f - k \left(\sum_{i=1}^n q_i^d \right) \right] q_i^d + \left[a - b \left(\sum_{i=1}^n q_i^d + \sum_{i=1}^n q_i^m \right) - F - k \left(\sum_{i=1}^n q_i^m \right) - t^e \right] q_i^m \quad (\text{A-6})$$

Differentiating (A-6) with respect to q_i^d and q_i^m yields the profit-maximizing first order conditions for firm i :

$$Q^d = \frac{\partial \pi_i}{\partial q_i^d} = \left[a - b \left(\sum_{i=1}^n q_i^d + \sum_{i=1}^n q_i^m \right) - f - k \left(\sum_{i=1}^n q_i^d \right) \right] + q_i^d (-b - k) + q_i^m (-b) = 0 \quad (\text{A-7})$$

$$Q^m = \frac{\partial \pi_i}{\partial q_i^m} = \left[a - b \left(\sum_{i=1}^n q_i^d + \sum_{i=1}^n q_i^m \right) - F - K \left(\sum_{i=1}^n q_i^m \right) - t^e \right] + q_i^m (-b - K) + q_i^d (-b) = 0 \quad (\text{A-8})$$

Since $q_i^d = q_j^d = q^d$ and $q_i^m = q_j^m = q^m$ for $i \neq j$ in a Cournot equilibrium, (A-7) and (A-8)

can be rewritten as:

$$Q^d = \frac{\partial \pi_i}{\partial q_i^d} = a - nbq^d - nbq^m - f - nkq^d - (b+k)q^d - bq^m = 0$$

$$Q^m = \frac{\partial \pi_i}{\partial q_i^m} = a - nbq^d - nbq^m - F - nKq^m - t^e - (b+k)q^m - bq^d = 0$$

Rearranging the above equations gives:

$$Q^d = \frac{\partial \pi_i}{\partial q_i^d} = (a - f) - (b+k)(n+1)q^d - b(n+1)q^m = 0 \quad (\text{A-9})$$

$$Q^m = \frac{\partial \pi_i}{\partial q_i^m} = (a - F - t^e) - b(n+1)q^d - (b+K)(n+1)q^m = 0 \quad (\text{A-10})$$

(A-9) and (A-10) can be written in a matrix form to solve simultaneous equations for q^d and

q^m :

$$\begin{bmatrix} (b+k)(n+1) & b(n+1) \\ b(n+1) & (b+K)(n+1) \end{bmatrix} \begin{bmatrix} q^d \\ q^m \end{bmatrix} = \begin{bmatrix} a - f \\ a - F - t^e \end{bmatrix} \quad (\text{A-11})$$

Then Cramer's rule yields:

$$q^d = \frac{\det \begin{bmatrix} a-f & b(n+1) \\ a-F-t^e & (b+K)(n+1) \end{bmatrix}}{\det \begin{bmatrix} (b+k)(n+1) & b(n+1) \\ b(n+1) & (b+K)(n+1) \end{bmatrix}} = \frac{(a-f)(b+K)(n+1) - b(n+1)(a-F-t^e)}{(b+k)(n+1)(b+K)(n+1) - b^2(n+1)^2} \quad (\text{A-12})$$

$$q^m = \frac{\det \begin{bmatrix} (b+k)(n+1) & a-f \\ b(n+1) & a-F-t^e \end{bmatrix}}{\det \begin{bmatrix} (b+k)(n+1) & b(n+1) \\ b(n+1) & (b+K)(n+1) \end{bmatrix}} = \frac{(b+k)(n+1)(a-F-t^e) - b(n+1)(a-f)}{(b+k)(n+1)(b+K)(n+1) - b^2(n+1)^2} \quad (\text{A-13})$$

Using (A-12) and (A-13), $Q_m(t^e)$ is finally obtained as:

$$Q_m(t^e) = nq^m = n \left[\frac{(b+k)(n+1)(a-F-t^e) - b(n+1)(a-f)}{(b+k)(n+1)(b+K)(n+1) - b^2(n+1)^2} \right] \quad (\text{A-14})$$

Case I

Exclusive rights to import are given to m number of firms. The remaining $n-m$ firms only operate in a domestic market. Domestic inverse demand function is (A-15), where Q^{de} is goods purchased domestically by m firms, Q^{me} is goods from import by m firms, and Q^d is goods purchased domestically by $n-m$ firms. The term e denotes firms with exclusive rights.

$$P = a - b(Q^{de} + Q^d + Q^{me}) \quad (\text{A-15})$$

Cost functions for domestic firms and firms with exclusive rights are respectively:

$$P_A = f + k(Q^{de} + Q^d) \quad (\text{A-16})$$

$$P_W = f + KQ^{me} \quad (\text{A-17})$$

With exclusive rights, firm i 's profit function becomes:

$$\begin{aligned}\pi_i &= (P - P_A)q_i^{de} + (P - P_W)q_i^{me} \\ \pi_i &= \left[a - b\left(\sum_{i=1}^m q_i^{de} + \sum_{j=m+1}^n q_j^d + \sum_{i=1}^m q_i^{me}\right) - f - k\left(\sum_{i=1}^m q_i^{de} + \sum_{j=m+1}^n q_j^d\right) \right] q_i^{de} \\ &+ \left[a - b\left(\sum_{i=1}^m q_i^{de} + \sum_{j=m+1}^n q_j^d + \sum_{i=1}^m q_i^{me}\right) - F - K\left(\sum_{i=1}^m q_i^{me}\right) \right] q_i^{me}\end{aligned}\quad (\text{A-18})$$

Differentiating (A-18) with respect to q_i^{me} and q_i^{de} yields the profit-maximizing first order conditions for firm i :

$$\frac{\partial \pi_i}{\partial q_i^{de}} = \left[a - b\left(\sum_{i=1}^m q_i^{de} + \sum_{j=m+1}^n q_j^d + \sum_{i=1}^m q_i^{me}\right) - f - k\left(\sum_{i=1}^m q_i^{de} + \sum_{j=m+1}^n q_j^d\right) \right] + (-b-k)q_i^{de} + (-b)q_i^{me} = 0 \quad (\text{A-19})$$

$$\frac{\partial \pi_i}{\partial q_i^{me}} = \left[a - b\left(\sum_{i=1}^m q_i^{de} + \sum_{j=m+1}^n q_j^d + \sum_{i=1}^m q_i^{me}\right) - F - K\left(\sum_{i=1}^m q_i^{me}\right) \right] + (-b)q_i^{de} + (-b-K)q_i^{me} = 0 \quad (\text{A-20})$$

Given a Cournot equilibrium assumption, (A-19) and (A-20) can be rewritten as:

$$\frac{\partial \pi_i}{\partial q_i^{de}} = \left[a - bmq^{de} - b(n-m)q^d - bmq^{me} - f - kmq^{de} - k(n-m)q^d \right] + (-b-k)q^{de} + (-b)q^{me} = 0$$

$$\frac{\partial \pi_i}{\partial q_i^{me}} = (-b)q^{de} + \left[a - bmq^{de} - b(n-m)q^d - bmq^{me} - F - Kmq^{me} \right] + (-b-K)q^{me} = 0$$

Rearranging the above equations yields:

$$\frac{\partial \pi_i}{\partial q_i^{de}} = (a - f) - (b+k)(m+1)q^{de} - (b+k)(n-m)q^d - b(m+1)q^{me} = 0 \quad (\text{A-21})$$

$$\frac{\partial \pi_i}{\partial q_i^{me}} = (a - F) - b(m+1)q^{de} - b(n-m)q^d - (b+K)(m+1)q^{me} = 0 \quad (\text{A-22})$$

Firm j 's profit function in absence of exclusive rights is shown by:

$$\pi_j = (P - P_A)q_j^d$$

$$\pi_j = \left[a - b \left(\sum_{i=1}^m q_i^{de} + \sum_{j=m+1}^n q_j^d + \sum_{i=1}^m q_i^{me} \right) - f - k \left(\sum_{j=m+1}^n q_j^d + \sum_{i=1}^m q_i^{de} \right) \right] q_j^d \quad (\text{A-23})$$

Differentiating (A-23) with respect to q_j^d yields the profit-maximizing first order condition:

$$\frac{\partial \pi_j}{\partial q_j^d} = a - bmq^{de} - b(n-m)q^d - bmq^{me} - f - k(n-m)q^d - kmq^{de} + (-b-k)q^d = 0 \quad (\text{A-24})$$

Rearranging the terms gives:

$$\frac{\partial \pi_j}{\partial q_j^d} = (a - f) - m(b+k)q^{de} - (b+k)(n-m+1)q^d - bmq^{me} = 0 \quad (\text{A-25})$$

(A-21), (A-22) and (A-25) can be written in a matrix form for a convenience to solve simultaneous equations for q^{de} , q^d and q^{me} .

$$\begin{bmatrix} (b+k)(m+1) & (b+k)(n-m) & b(m+1) \\ b(m+1) & b(n-m) & (b+k)(m+1) \\ m(b+k) & (b+k)(n-m+1) & bm \end{bmatrix} \begin{bmatrix} q^{de} \\ q^d \\ q^{me} \end{bmatrix} = \begin{bmatrix} a-f \\ a-f \\ a-f \end{bmatrix} \quad (\text{A-26})$$

Cramer's rule yields:

$$q^{me} = \frac{\det \begin{bmatrix} (b+k)(m+1) & (b+k)(n-m) & a-f \\ b(m+1) & b(n-m) & a-F \\ m(b+k) & (b+k)(n-m+1) & a-f \end{bmatrix}}{\det \begin{bmatrix} (b+k)(m+1) & (b+k)(n-m) & b(m+1) \\ b(m+1) & b(n-m) & (b+K)(m+1) \\ m(b+k) & (b+k)(n-m+1) & bm \end{bmatrix}} \quad (\text{A-27})$$

As before, t^e under the Case I is set as:

$$Q_m(t^e) = Q_{m,casel}^{STE} \quad (\text{A-28})$$

Solving (A-14) and (A-28) in terms of t^e shows:

$$t_{casel}^e = \frac{1}{\phi_1} \left\{ \left[\phi_1 - \frac{\Omega_1 m}{\Omega_2 n} (b+k) \right] (a-F) - \left[b(n+1) - \frac{\Omega_1 m}{\Omega_2 n} b \right] (a-f) \right\} \quad (\text{A-29})$$

where, $\phi_1 = (b+k)(n+1)$, $\phi_2 = (b+K)(n+1)$, $\Omega_1 = \phi_1 \phi_2 - b^2 (n+1)^2$ and $\Omega_2 = (m+1)[(b+k)(b+K) - b^2]$

Case II

STE has exclusive rights only for imports. In a domestic market, there are m firms using STE and $n-m$ remaining firms. A Cournot competition is assumed as before. STE's objective is to maximize producer surplus while other firms maximize their own profits.

Domestic inverse demand function is given by:

$$P = a - b(Q^{de} + Q^d + Q^{me}) \quad (\text{A-30})$$

Cost functions for domestic and import goods are respectively:

$$P_A = f + k(Q^{de} + Q^d) \quad (\text{A-31})$$

$$P_W = F + KQ^{me} \quad (\text{A-32})$$

STE's objective function is:

$$\begin{aligned} W &= PQ^{de} - \int_0^{Q^{de}} P_A dQ^{de} + \pi^{me} \\ &= PQ^{de} - \int_0^{Q^{de}} P_A dQ^{de} + (P - P_w)Q^{me} \end{aligned}$$

$$\begin{aligned} &= \left[a - b(Q^{de} + \sum_{j=1}^{n-m} q_j^d + Q^{me}) \right] Q^{de} - \int_0^{Q^{de}} \left[f + k(Q^{de} + \sum_{j=1}^{n-m} q_j^d) \right] dQ^{de} \\ &+ \left[a - b(Q^{de} + \sum_{j=1}^{n-m} q_j^d + Q^{me}) - F - KQ^{me} \right] Q^{me} \end{aligned} \quad (\text{A-33})$$

Differentiating (A-33) with respect to Q^{me} and Q^{de} yields producer surplus-maximizing first order conditions for STE:

$$\frac{\partial W}{\partial Q^{de}} = \left[a - b(Q^{de} + \sum_{j=1}^{n-m} q_j^d + Q^{me}) \right] + (-b)Q^{de} - f - k(Q^{de} + \sum_{j=1}^{n-m} q_j^d) + (-b)Q^{me} = 0 \quad (\text{A-34})$$

$$\frac{\partial W}{\partial Q^{me}} = (-b)Q^{de} + \left[a - b(Q^{de} + \sum_{j=1}^{n-m} q_j^d + Q^{me}) - F - KQ^{me} \right] + (-b - K)Q^{me} = 0 \quad (\text{A-35})$$

Under a Cournot equilibrium, (A-34) and (A-35) can be rewritten as:

$$\begin{aligned} \frac{\partial W}{\partial Q^{de}} &= a - bQ^{de} - b(n-1)q^d - bQ^{me} - bQ^{de} - f - kQ^{de} - k(n-1)q^d - bQ^{me} = 0 \\ \frac{\partial W}{\partial Q^{me}} &= -bQ^{de} + a - bQ^{de} - b(n-1)q^d - bQ^{me} - F - KQ^{me} - (b + K)Q^{me} = 0 \end{aligned}$$

Rearranging the above equations yields:

$$\frac{\partial W}{\partial Q^{de}} = (a - f) - (2b + k)Q^{de} - (b + k)(n - 1)q^d - 2bQ^{me} = 0 \quad (\text{A-36})$$

$$\frac{\partial W}{\partial Q^{me}} = (a - F) - 2bQ^{de} - b(n - 1)q^d - 2(b + K)Q^{me} = 0 \quad (\text{A-37})$$

Without exclusive rights, firm j 's profit function is given by:

$$\begin{aligned} \pi_j &= (P - P_A)q_j^d \\ \pi_j &= \left[a - b(Q^{de} + \sum_{j=1}^{n-m} q^d + Q^{me}) - f - k(Q^{de} + \sum_{j=1}^{n-m} q_j^d) \right] q_j^d \end{aligned} \quad (\text{A-38})$$

Differentiating (A-38) with respect to q_j^d yields the profit-maximizing first order condition

for firm j :

$$\frac{\partial \pi_j}{\partial q_j^d} = \left[a - b(Q^{de} + \sum_{j=1}^{n-m} q^d + Q^{me}) - f - k(Q^{de} + \sum_{j=1}^{n-m} q_j^d) \right] + (-b - k)q_j^d \quad (\text{A-39})$$

The Cournot equilibrium condition yields:

$$\frac{\partial \pi_j}{\partial q_j^d} = \left[a - b(Q^{de} + \sum_{j=1}^{n-m} q^d + Q^{me}) - f - k(Q^{de} + \sum_{j=1}^{n-m} q_j^d) \right] + (-b - k)q_j^d = 0$$

$$\frac{\partial \pi_j}{\partial q_j^d} = a - bQ^{de} - b(n - 1)q^d - bQ^{me} - f - kQ^{de} - k(n - 1)q^d - (b + k)q^d = 0$$

And thus,

$$\frac{\partial \pi_j}{\partial q_j^d} = (a - f) - (b + k)Q^{de} - n(b + k)q^d - bQ^{me} = 0 \quad (\text{A-40})$$

(A-36), (A-37) and (A-40) can be written in a matrix form to solve simultaneous equations

for Q^{de} , Q^{me} and q^d :

$$\begin{bmatrix} (2b+k) & (b+k)(n-m) & 2b \\ 2b & b(n-m) & 2(b+K) \\ (b+k) & n(b+k) & b \end{bmatrix} \begin{bmatrix} Q^{de} \\ q^d \\ Q^{me} \end{bmatrix} = \begin{bmatrix} a-f \\ a-F \\ a-f \end{bmatrix} \quad (\text{A-41})$$

Cramer's rule yields:

$$Q^{me} = \frac{\det \begin{bmatrix} (2b+k) & (b+k)(n-m) & a-f \\ 2b & b(n-m) & a-F \\ (b+k) & n(b+k) & a-f \end{bmatrix}}{\det \begin{bmatrix} (2b+k) & (b+k)(n-m) & 2b \\ 2b & b(n-m) & 2(b+K) \\ (b+k) & n(b+k) & b \end{bmatrix}} \quad (\text{A-42})$$

$$= \frac{b^2 nF + bnFk - abnk - 2bfk - b^2 nf + b^2 f + 2bkF}{-4bkK - 3b^2 nk - 2b^2 nK - b^2 k - 2bnKk - 2nb^3 - 2b^2 K - 2k^2 b - 2k^2 - 2k^2 K} \quad (\text{A-43})$$

Total volume of imports is determined by:

$$Q_{m,case2}^{STE} = Q^{me} \quad (\text{A-44})$$

The t^e can be derived from the relationship:

$$Q_m(t^e) = Q_{m,case2}^{STE} \quad (\text{A-45})$$

Therefore, it becomes:

$$n \left[\frac{(b+k)(n+1)(a-F-t^e) - b(n+1)(a-f)}{(b+k)(n+1)(b+K)(n+1) - b^2(n+1)^2} \right] = \frac{b^2nF + bnFk - abnk - 2bfk - b^2nf + b^2f + 2bkF}{-4bkK - 3b^2nk - 2b^2nK - b^2k - 2bnKk - 2nb^3 - 2b^2K - 2k^2b - 2k^2 - 2k^2K} \quad (\text{A-46})$$

Solving (A-46) in terms of t^e yields:

$$t_{case2}^e = \frac{1}{\phi_1} \left\{ \phi_1(a-F) - \left[\frac{\Omega_1}{n} \Phi_1 + b(n+1)(a-f) \right] \right\} \quad (\text{A-47})$$

where, $\phi_1 = (b+k)(n+1)$, $\phi_2 = (b+K)(n+1)$, $\Omega_1 = \phi_1\phi_2 - b^2(n+1)^2$, and

$$\Phi_1 = \frac{b^2nF + bnFk - abnk - 2bfk - b^2nf + b^2f + 2bkF}{-4bkK - 3b^2nk - 2b^2nK - b^2k - 2bnKk - 2nb^3 - 2b^2K - 2k^2b - 2k^2 - 2k^2K}$$

Case III

STE's objective is to maximize consumer surplus while other firms maximize its own profits.

Domestic inverse demand function is given by (A-48), where Q^{de} , Q^{me} and Q^d are the same as before.

$$P = a - b(Q^{de} + Q^d + Q^{me}) \quad (\text{A-48})$$

Cost functions for domestic and import goods are respectively given by:

$$P_A = f + k(Q^{de} + Q^d) \quad (\text{A-49})$$

$$P_W = F + KQ^{me} \quad (\text{A-50})$$

STE's objective function is:

$$\begin{aligned}
W &= \int_0^Q PdQ - PQ + \pi^{de} + \pi^{me} \\
&= \int_0^Q PdQ - PQ + (P - P_A)Q^{de} + (P - P_W)Q^{me} \\
&= \int_0^Q \left[a - b(Q^{de} + \sum_{j=1}^{n-m} q^d + Q^{me}) \right] dQ - \left[a - b(Q^{de} + \sum_{j=1}^{n-m} q^d + Q^{me}) \right] (Q^{de} + \sum_{j=1}^{n-1} q^d + Q^{me}) \\
&\quad + \left[a - b(Q^{de} + \sum_{j=1}^{n-m} q^d + Q^{me}) - f - k(Q^{de} + \sum_{j=1}^{n-1} q^d) \right] Q^{de} \\
&\quad + \left[a - b(Q^{de} + \sum_{j=1}^{n-m} q^d + Q^{me}) - F - KQ^{me} - F - KQ^{me} \right] Q^{me}
\end{aligned} \tag{A-51}$$

Differentiating (A-51) with respect to Q^{me} and Q^{de} yields the consumer surplus-maximizing first order conditions for STE:

$$\frac{\partial W}{\partial Q^{de}} = (a - f) - (b + 2k)Q^{de} - k(n-1)q^d - bQ^{me} = 0 \tag{A-52}$$

$$\frac{\partial W}{\partial Q^{me}} = (a - F) - 2bQ^{de} - b(n-1)q^d - (b + 2K)Q^{me} = 0 \tag{A-53}$$

Firm j 's profit function is:

$$\begin{aligned}
\pi_j &= (P - P_A)q_j^d \\
\pi_j &= \left[a - b(Q^{de} + \sum_{j=1}^{n-m} q^d + Q^{me}) - f - k(Q^{de} + \sum_{j=1}^{n-m} q_j^d) \right] q_j^d
\end{aligned} \tag{A-54}$$

Differentiating (A-54) with respect to q_j^d yields profit-maximizing first order condition for firm j :

$$\frac{\partial \pi_j}{\partial q_j^d} = \left[a - b(Q^{de} + \sum_{j=1}^{n-m} q^d + Q^{me}) - f - k(Q^{de} + \sum_{j=1}^{n-m} q_j^d) \right] + (-b - k)q_j^d \tag{A-55}$$

Application of a Cournot equilibrium provides:

$$\frac{\partial \pi_j}{\partial q_j^d} = \left[a - b(Q^{de} + \sum_{j=1}^{n-m} q_j^d + Q^{me}) - f - k(Q^{de} + \sum_{j=1}^{n-m} q_j^d) \right] + (-b-k)q_j^d = 0$$

$$\frac{\partial \pi_j}{\partial q_j^d} = a - bQ^{de} - b(n-1)q^d - bQ^{me} - f - kQ^{de} - k(n-1)q^d - (b+k)q^d = 0$$

Rearranging the above terms gives:

$$\frac{\partial \pi_j}{\partial q_j^d} = (a-f) - (b+k)Q^{de} - n(b+k)q^d - bQ^{me} = 0 \quad (\text{A-56})$$

(A-52), (A-53) and (A-56) can be written in a matrix form to solve out Q^{de} , Q^{me} and q^d :

$$\begin{bmatrix} (b+2k) & k(n-m) & 2b \\ 2b & b(n-m) & (b+2K) \\ (b+k) & n(b+k) & b \end{bmatrix} \begin{bmatrix} Q^{de} \\ q^d \\ Q^{me} \end{bmatrix} = \begin{bmatrix} a-f \\ a-F \\ a-f \end{bmatrix} \quad (\text{A-57})$$

Cramer's rule yields:

$$Q^{me} = \frac{\det \begin{bmatrix} (b+2k) & k(n-1) & a-f \\ 2b & b(n-1) & a-F \\ (b+k) & n(b+k) & a-f \end{bmatrix}}{\det \begin{bmatrix} (b+2k) & k(n-m) & b \\ 2b & b(n-m) & (b+2K) \\ (b+k) & n(b+k) & b \end{bmatrix}} \quad (\text{A-58})$$

$$= \frac{ab^2n - abnk + b^2nF + 2bnFk - ank^2 + k^2nF - 2b^2nf - bnfk + kbF - ak^2 + k^2F - bkf}{b^3n - b^2nk - 2b^2nK - 4bnkK - bnk^2 - 2nk^2K - 2bkK - bk^2 - 2k^2K} \quad (\text{A-59})$$

From (A-14) and (A-59), t^e can be derived from the relationship:

$$Q_m(t^e) = Q_{m,case3}^{STE} \quad (\text{A-60})$$

And thus,

$$n \left[\frac{(b+k)(n+1)(a-F-t^e) - b(n+1)(a-f)}{(b+k)(n+1)(b+K)(n+1) - b^2(n+1)^2} \right] = \frac{ab^2n - abnk + b^2nF + 2bnFk - ank^2 + k^2nF - 2b^2nf - bnfk + kbF - ak^2 + k^2F - bkf}{b^3n - b^2nK - 2b^2nk - 4bnkK - bnk^2 - 2nk^2K - 2bkK - bk^2 - 2k^2K} \quad (\text{A-61})$$

Solving (A-14) and (A-61) in terms of t^e shows:

$$t_{case3}^e = \frac{1}{\phi_1} \left\{ \phi_1(a-F) - \left[\frac{\Omega_1}{n} \Phi_2 + b(n+1)(a-f) \right] \right\} \quad (\text{A-62})$$

where, $\phi_1 = (b+k)(n+1)$, $\phi_2 = (b+K)(n+1)$, $\Omega_1 = \phi_1\phi_2 - b^2(n+1)^2$ and

$$\Phi_2 = \frac{ab^2n - abnk + b^2nF + 2bnFk - ank^2 + k^2nF - 2b^2nf - bnfk + kbF - ak^2 + k^2F - bkf}{b^3n - b^2nk - 2b^2nK - 4bnkK - bnk^2 - 2nk^2K - 2bkK - bk^2 - 2k^2K}$$